MATH632: INTRODUCTION TO STOCHASTIC PROCESSES

Fall 2021

COURSE INFORMATION

Instructor	Hanbaek Lyu (Email: *hlyu@math.wisc.edu*, Office: Van Vleck 303)
Lectures	TTh 1:00PM - 2:15PM at Van Vleck B139
Office hours	(tentative) M 2PM - 4PM
Textbook	Rick Durrett, *Essentials of Stochastic Processes*, 2nd edition. (Free download)
Canvas	https://canvas.wisc.edu/courses/262022
Piazza	https://piazza.com/wisc/fall2021/ba33
Grader	

GENERAL INFORMATION

Instructional Modality	In-person
Credits	3.0. The three credit hours are met by two 75-minute class periods and a for about 2-3 hours outside of class student work for every class period.
Course Designations	Breadth - Natural Science Level - Advanced L&S Credit - Counts as Liberal Arts and Science credit in L&S Grad 50% - Counts toward 50% graduate coursework requirement

Regular and Substantive Student-Instructor Interaction

Participation in regularly scheduled learning sessions (where there is an opportunity for direct interaction between the student and the qualified instructor), office hours.

COURSE DESCRIPTION

A stochastic process is a sequence of random variables, evolving in time according to some underlying mechanism that is designed to capture some natural phenomena. Topics include discrete-time Markov chains, Martingales, Poisson point processes, continuous-time Markov chains, and renewal processes. Applications to queueing, branching, and other models in science, engineering and business.

REQUISITS

(MATH/STAT 431, 309, STAT 311 or MATH 531) and (MATH 320, 340, 341, 375, 421 or 531) or graduate/professional standing or member of the Pre-Masters Mathematics (Visiting International) Program.

Less formally, it is important to have a good knowledge of undergraduate probability. This means familiarity with basic probability models, random variables and their distributions, expectations, joint distributions, independence, conditional probabilities, the law of large numbers and the central limit theorem. If you wish to acquire a book for review, the Math 431 course book Introduction to Probability by David Anderson, Timo Seppäläinen, and Benedek Valkó is recommended.

COURSE LEARNING OUTCOMES

Students will be able to

- recall and state the formal definitions of the mathematical objects and their properties for stochastic processes (e.g., discrete space Markov chains, Poisson processes, renewal processes, branching processes, etc.).
- use such definitions to argue that a mathematical object does or does not have the condition of being a particular type or having a particular property (e.g., irreducibility, aperiodicity, recurrence, transience, the Markov property, etc.).
- recall and state the standard theorems of stochastic processes. (e.g., laws of large numbers for Markov chains, existence of limiting/stationary distributions, law of large numbers for renewal processes, etc.) and recall the arguments for these theorems and the underlying logic of their proofs.
- construct mathematical arguments related to the above definitions, properties, and theorems, including the construction of examples and counterexamples.
- convey arguments in oral and written forms using English and appropriate mathematical terminology, notation and grammar.
- model simple real life situations by means of discrete-space stochastic processes and calculate probabilities associated with those processes.

GRADING AND EXAMS

Grading Scheme Participation (5%) + Homework (20%) + Midterm 1 (20%) + Midterm 2 (20%) + Final (35%)
(You should attend the final exam to pass the course.)

Grade Cutoffs The following grade lines are guaranteed in advance. A percentage score in the indicated range guarantees at least the letter grade next to it:

A: [100,90], AB: (90,87], B: (87,76], BC: (76,74], C: (74,62], D: (62,50]

Final letter grades are not curved but the grade lines above may be lowered at the end. Class attendance is not part of the grading.

Some weight from early exams could be shifted to later ones in certain special cases (but only if that helps the student.)

Midterm 1 Wednesday, 10/13, 7:15PM-8:45PM (Location TBD)

Midterm 2 Wednesday, 11/17, 7:15PM-8:45PM (Location TBD)

Final Saturday, 12/18, 7:45AM - 9:45AM (Location TBD)

Past Exams Here are the old exams from the Library: LINK

PARTICIPATION

- Each student should set up a 15-min meeting (either in-person or on zoom) with the instructor and present a solution to *any* problem in the textbook or in the provided lecture notes.
- Deadline to set up a course participation meeting is **Nov. 5 (week 9).**

HOMEWORK

- Homework problems will be posted every Thursday under "Assignments" in Canvas and will be due the following **Wed. 10 pm**. The lowest homework score will be dropped.

- Homework will be handed in, graded, and returned using Canvas. It should be uploaded to Canvas in the format of a **single PDF file**. Put your problems in the correct order (to simplify this, it might be useful to write each problem on a separate sheet of paper (certainly if you do not type your solutions). Please also make sure all pages are in the right orientation when you convert them. Do not hand in your rough draft or first attempt. Papers that are unreadable or disorganized cannot be graded. It is a good habit to download your submission every time and check everything is fine.
- The homework is graded according to its correctness, completeness and presentation. Answers alone carry no credit. One should provide clear arguments and steps that lead to your solution/conclusion. Organize your work neatly. Use proper English. Answers should be simplified as much as possible.
- If you have mathematical questions about HW, I strongly encourage you to use Piazza (you may use anonymous option if you wish). You certainly can write me an e-mail with the questions, but Piazza is much more useful for mathematical formulas than e-mail programs.
- Observe rules of academic integrity. Handing in plagiarized work, whether copied from a fellow student or off the web, is not acceptable. Plagiarism cases will lead to sanctions. You can discuss the problems, ideas, hints with your classmates, however, you should always write down the solutions on your own.

CAMPUS SPACES FOR VIRTUAL LEARNING & TESTING

Dedicated on-campus spaces with high-speed internet are available for students to reserve (see LINK) for any exam/quiz taken during the semester. Computers can also be requested.

COURSE EVALUATION

UW-Madison now uses an online course evaluation survey tool, AEFIS. You will receive an official email two weeks prior to the end of the semester when your course evaluation is available. You will receive a link to log into the course evaluation with your NetID where you can complete the evaluation and submit it, anonymously. Your participation is an integral component of this course, and your feedback is important to me. I strongly encourage you to participate in the course evaluation. You are also very welcome to provide a feedback about the course during the semester (as a direct e-mail to me, or, if you prefer the anonymous option, you can use anonymous comments on Piazza).

ACADEMIC INTEGRITY

By enrolling in this course, each student assumes the responsibilities of an active participant in the UW-Madison community of scholars, in which everyones academic work and behavior are held to the highest academic integrity standards. Academic misconduct compromises the integrity of the university. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these acts are examples of academic misconduct, which can result in disciplinary action. This includes but is not limited to failure on the assignment/course, disciplinary probation, or suspension. Substantial or repeated cases of misconduct will be forwarded to the Office of Student Conduct and Community Standards for additional review. For more information, refer to LINK.

ACADEMIC CALENDAR& RELIGIOUS OBSERVANCES

See LINK.

ACCOMMODATIONS FOR STUDENTS WITH DISABILITIES

The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (Faculty Document 1071) require that students with disabilities be reasonably accommodated in instruction and campus life. Reasonable accommodations for students with disabilities is a shared faculty and student responsibility. Students are expected to inform the instructor of their need for instructional accommodations by the end of the third week of the semester, or as soon as possible after a disability has been incurred or recognized. The instructor will work either directly with you or in coordination with the McBurney Center to identify and provide reasonable instructional accommodations. Disability information, including instructional accommodations as part of a student's educational record, is confidential and protected under FERPA. For more information, refer to LINK.

DIVERSITY AND INCLUSION

Diversity is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals.

The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background – people who as students, faculty, and staff serve Wisconsin and the world. For more information, refer to LINK.

RULES, RIGHTS & RESPONSIBILITIES

During the global COVID-19 pandemic, we must prioritize our collective health and safety to keep ourselves, our campus, and our community safe. As a university community, we must work together to prevent the spread of the virus and to promote the collective health and welfare of our campus and surrounding community. For more information, refer to LINK.

UW-MADISON BADGER PLEDGE

See LINK.

PRIVACY OF STUDENT INFORMATION & DIGITAL TOOLS: TEACHING & LEARNING ANALYTICS & PROCTORING STATEMENT

The privacy and security of faculty, staff and students personal information is a top priority for UW-Madison. The university carefully reviews and vets all campus-supported digital tools used to support teaching and learning, to help support success through learning analytic, and to enable proctoring capabilities. UW-Madison takes necessary steps to ensure that the providers of such tools prioritize proper handling of sensitive data in alignment with FERPA, industry standards and best practices. Under the Family Educational Rights and Privacy Act (FERPA which protects the privacy of student education records), student consent is not required for the university to share with school officials those student education records necessary for carrying out those university functions in which they have legitimate educational interest. 34 CFR 99.31(a)(1)(i)(B). FERPA specifically allows universities to designate vendors such as digital tool providers as school officials, and accordingly to share with them personally identifiable information from student education records if they perform appropriate services for the university and are subject to all applicable requirements governing the use, disclosure and protection of student data.

PRIVACY OF STUDENT RECORDS AND THE USAGE OF AUDIO RECORDED LECTURES

Lecture materials and recordings for this course are protected intellectual property at UW-Madison. Students in this course may use the materials and recordings for their personal use related to participation in this class. Students may also take notes solely for their personal use. If a lecture is not already recorded, you are not authorized to record my lectures without my permission unless you are considered by the university to be a qualified student with a disability requiring accommodation. [Regent Policy Document 4-1] Students may not copy or have lecture materials and recordings outside of class, including posting on internet sites or selling to commercial entities. Students are also prohibited from providing or selling their personal notes to anyone else or being paid for taking notes by any person or commercial firm without the instructors express written permission. Unauthorized use of these copyrighted lecture materials and recordings constitutes copyright infringement and may be addressed under the universitys policies, UWS Chapters 14 and 17, governing student academic and non-academic misconduct.

CAMPUS GUIDANCE ON THE USE OF FACE COVERINGS

Students of the class are expected to comply with the Universitys current COVID rules and policies that are maintained here: (see in particular link).

Face coverings must be correctly worn on campus at all times and in all places (both outside and inside), except by students in their assigned residence hall rooms; by employees when alone in a private, unshared lab or office; when traveling alone in a private vehicle; and when exercising outside in a way that maintains 6 feet of distance from other people.

Students with disabilities or medical conditions who are unable to wear a face covering should contact the McBurney Disability Resource Center or their Access Consultant if they are already affiliated. Students requesting an accommodation unrelated to disability or medical condition, should contact the Dean of Students Office.

Students who choose not to wear a face covering may not attend in-person classes, unless they are approved for an accommodation or exemption. All other students not wearing a face covering will be asked to put one on or leave the classroom. Students who refuse to wear face coverings appropriately or adhere to other stated requirements will be reported to the Office of Student Conduct and Community Standards and will not be allowed to return to the classroom until they agree to comply with the face covering policy. An instructor may cancel or suspend a course in-person meeting if a person is in the classroom without an approved face covering in position over their nose and mouth and refuses to immediately comply.

QUARANTINE OR ISOLATION DUE TO COVID-19

Students should continually monitor them selves for COVID-19 symptoms and get tested for the virus if they have symptoms or have been in close contact with someone with COVID-19. Students should reach out to instruc tors as soon as possible if they become ill or need to isolate or quarantine, in order to make alternate plans for how to proceed with the course. Students are strongly encouraged to communicate with their instructor concerning their illness and the anticipated extent of their absence from the course (either in-person or remote). The instructor will work with the student to provide alternative ways to complete the course work.

TENTATIVE COURSE SCHEDULE

Below is a tentative course schedule. There could be a slight change depending on our progress.

Week	Date	Topics	c.f.
1	Th 9/9	Review of basic probability, SLLN, Arrival processes, renewal processes	
2	T 9/14	Renewal reward processes, Renewal SLLN, Renewal Reward SLLN	
	Th 9/16	Definition of Markov chains. Examples: the simple random walk, gamblers ruin, two state Markov chain, success run	HW 0
3	T 9/21	multistep transition probabilities, connection to the powers of the transition probability matrix, Stopping times	
	Th 9/23	The strong Markov property, recurrence and transience	HW 1
4	T 9/28	Invariant/stationary distribution and invariant measure for a Markov chain	
	Th 9/30	RW on graphs and its stationary distribution, the connection between the invariant distribution and the expected return time the renewal chain	HW 2
5	T 10/5	Long time behavior of Markov Chains, the asymptotic frequency of returns for a recurrent state	
	Th 10/7	The Strong Law of Large Numbers for irreducible positive recurrent Markov Chains	HW 3
6	T 10/12	Aperiodicity of MCs, convergence of irreducible aperiodic MCs to its unique stationary distribution (pf by coupling, pf by transition matrix)	Midterm (10/13)
	Th 10/14	MCMC	
7	T 10/19	The definition of a martingale, examples: sum of iid mean zero random variables, product of iid mean one random variables	
	Th 10/21	Binomial model in math finance and no arbitrage principle, the optional stopping theorem	HW 4
8	T 10/26	The martingale limit theorem, the limiting distribution in the Polya urn, waiting for patterns in a sequence of coin flips	
	Th 10/28	Moment generating functions, branching processes, extinction probability in the subcritical, critical and supercritical cases	HW 5
9	T 11/2	The branching process as a Markov Chain. The transition probability function. Transience and recurrence of states	
	Th 11/4	Application of the martingale limit theorem in the supercritical case, the Poisson point process as a renewal process	HW 6
10	T 11/9	Definitions of Poisson processes, Memoryless property, stopping times	
	Th 11/11	Thinning and superposition of PPs, M/M/1 queue	HW 7

Week	Date	Topics	c.f.
11	T 11/16	PPs as counting processes, non-homogeneous PPs	Midterm 2 (11/17)
	Th 11/18	Little's Law and queuing theory	
12	T 11/23	Limit in distribution for age and residual life for general renewal processes, Continuous time Markov chains: the Markov property, transition probability function the Poisson process as a Markov chain	HW 8
	Th 11/25	Thanksgiving	
13	T 11/30	Constructions of the continuous time Markov chain (CTMC) – MC + exponential holding times, graphical construction with Poisson clocks	
	Th 12/2	Gillespie algorithm, the infinitesimal rates of a continuous time MC	HW 9
14	T 12/7	The Kolmogorov forward and backward equations, stationary distribution of CTMC	
	Th 12/9	CTMC: recurrence, transience, expected return times, reversible distributions, birth and death processes	HW 10
15	T 12/14	Review	
	Sat. 12/18	Final	